New Applications of the Principal Partition of Graphs to Electrical Network Analysis (Graphs and Combinatorics III)

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NEW APPLICATIONS OF THE PRINCIPAL PARTITION OF GRAPHS TO ELECTRICAL NETWORK ANALYSIS

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I. INTRODUCTION

The principal partition of a graph was defined by Kishi and Kajitani.(1) It was introduced in order to prove the validity of an algorithm for finding a pair of trees* in a graph which have as few edges as possible in common. Its first application to electrical network analysis was found a few months after its birth by Kishi and Kajitani themselves and a group of Nippon Electric Company.(2) The application is concerned with the minimum number of equations for the mixed analysis of electrical networks. This minimum number is called the topological degree of freedom of a network. The concept of the principal partition was found in the decomposition of a matrix by Iri.(3)(4) He related it to the decomposition by Dulmage and Mendelsohn. It was also extended to that of a matroid in a more detailed form by Tomizawa(5) and independently by Narayanan.(6)

In 1972 the author was trying to derive a set of state equations for an electrical networs by a graphical method. At that time he found some examples to which the method did not work:

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* A tree means a spanning tree.

Actually there can exist no state equations for the examples.[7] The reason for the non-existence is the lack of common trees of the current graph and the voltage graph. The existence of a common tree turns out to be a necessary condition for the existence of a unique solution of network equations.[8][9] In connection with the existence of a common tree of 2-graphs the author introduced the principal partition of 2-graphs(current and voltage graphs). A more detailed discussion of the principal partition will be found elsewhere in this issue.[10]

The new applications of the principal partition discussed here are those which can be related to the solvability problem mentioned above. The results obtained concerning the principal partition of 2-graphs are applicable to the network problems defined here. The problem themselves are of Menger type. The partition of graphs defined in connection with Menger's theorem in matroids[11] is shown to have a one-to-one correspondence with the principal partition of 2-graphs.

II. DIAGNOSIS AND SEQUENTIOAL ANALYSIS OF ELECTRICAL NETWORKS

The linear active network to be considered here is represented by 2-graphs, that is, the current graph G_i and the voltage graph G_v. The current[voltage]* graph represents the relations among the currents[voltages] in the network. Kirchhoff's current law (KCL)[Kirchhoff's voltage law(KVL)] is applied to G_i[G_v] to get network equations. G_i[G_v] is derived from the network as follows. First G is the graph obtained from the network by replac-

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* A dual sentence is obtained by replacing the words just before [ ] with those in [ ].

ing elements with edges. Then $G_i[G_v]$ is obtained from G by contracting[deleting] the edges corresponding to dependent voltage[current] sources and norators, and deleting[contracting] the edges corresponding to voltage[current] sensors and nullators. An edge in G_i and an edge in G_v which correspond to an element in the network, are considered to be the same edge. A voltage[current] sensor and a dependent current[voltage] source always form a pair, and an edge corresponding to only one of them is left in G_i or G_v. The edge represents the pair. Thus G_i and G_v have a common edge set, which is denoted by E. It is assumed that there is no special relation among the element values. Therefore the results obtained here are of a topological nature. Moreover it is assumed that G_i and G_v have at least one common tree and the network has a unique solution.

For any graph G, its edge set E and a subset E_s of E, we denote by $G \cdot E_s[G \times E_s]$ the graph obtained from G by deleting [contracting] the edges of $E-E_s$. The rank of G is denoted by r(G), and the nullity, by n(G). For any set A, $|A|$ denotes the cardinality of A. $\oplus$ denotes the union of edge-disjoint sets.

Diagnosability of linear active networks[12] It may happen in an electrical network that the resistances, capacitances and/or inductances, etc.(called element values) change in a lapse of time or by some other reasons, or there are stray elements whose element values are unknown. The diagnosis of a network is to detect such faults by determining currents and/or voltages of desired elements from the measured currents and voltages of certain other elements. E is partitioned into sets E_m, E_k and

E_u. E_m is further partitioned into E_b, E_j and E_e.

E_b: set of edges whose currents and voltages are both measurable,

E_j: set of edges whose currents only are measurable,

E_e: set of edges whose voltages only are measurable,

$E_m = E_b \oplus E_j \oplus E_e$,

E_k: set of edges whose element values are known,

E_u: set of edges whose element values are unknown,

F_i: set of edges whose currents are required to determine,

F_v: set of edges whose voltages are required to determine.

If the current[voltage] of an edge is measured and its element value is known, then its voltage[current] can be determined. Therefore such an edge is included in E_b, even if only its current[voltage] is actually measurable. As for an independent source, its voltage[current] cannot be determined from its current[voltage] only. Therefore, if its current[voltage] only can be measured or known, it is included in E_j[E_e], and if neither current nor voltage is measurable or known, it is included in E_u.

The network is said to be diagnosable if the required currents of the edges in F_i and the voltages of the edges in F_v can be all determined from the measured currents and/or voltages of the edges in E_m. If the current and voltage of an edge can [cannot] be determined from the measurements, it is called a determinate[indeterminate] edge.

Sequential Network Analysis[13] A sequential method of network analysis was defined by Moad.[14] The unknown currents and voltages in the network are sequentially related to properly chosen independent variable by use of KCL or KVL. At the end of this sequential process, a set of simultaneous equations called constraint equation, are obtained. These equations are then solved to determine independent variables. The detail of the sequential process is given by Algorithm 1 below. It is a modified version of Moad's method. For the simplicity of description, the treatment of independent sources is omitted.

E_j: set of edges corresponding to independent current sources,

E_e: set of edges corresponding to independent voltage sources,

$E_p \equiv E-E_j-E_e$.

$G_{ip} \equiv G_i \cdot (E_e \oplus E_p) x E_p$, $G_{vp} \equiv G_v \cdot (E_e \oplus E_p) x E_p$.

The edges associated with the independent variables are called independent edges.

ALGORITHM 1(Sequential process)

Step 0. $E_\beta \leftarrow \emptyset$($\emptyset$: null set)

Step 1. Choose, as the initial set of independent edges, a set of edges, denoted by E_{bi}, which contains neither cutset in G_{ip} nor tieset in G_{vp}. $E_\beta \leftarrow E_{bi}$.

Step 2. If $E_\beta = E_p$, stop.

Step 3. If an edge, e, in $E_p - E_\beta$ forms a cutset in G_{ip} with some edges in E_β, then add e to E_β, and go to Step 2.

Step 4. If an edge, e, in $E_p - E_\beta$ forms a tieset in G_{vp} with some edges in E_β, then add e to E_β, and go to Step 2.

Step 5. Choose, as an additional independent edge, an edge, e, in $E_p - E_\beta$. Add e to E_β, and go to Step 3.

If an edge is added to E_β, it is said to be covered. An edge in $E_p - E_\beta$ which forms a cutset[tieset] in G_{ip}[G_{vp}] with some of of the edges in E_β is called a current[voltage] dependent edge. Edge e in Step 3 is a current dependent edge. It may also be voltage dependent. If so, e is called a constraint edge. Edge e in Step 4 is a voltage dependent edge. If an edge is either current dependent or voltage dependent, but not both, then it is called a single-dependent edge.

The current[voltage] of a current[voltage] dependent edge can be given, by applying KCL[KVL] to the cutset[tieset], in terms of the currents[voltages] of edges in E_β. The current and the voltage of an edge are related by Ohm's law. Now at the beginning of the sequential process, $E_\beta=E_{bi}$. Then if a current or voltage dependent edge is added to E_β, its current and voltage can be given in terms of the independent variables. This can be repeated as Algorithm 1 proceeds, and all the currents and the voltages in the network can be given in terms of the independent variables. The current and the voltage of a constraint edge can be related independently to the independent variables. Then Ohm's law for the edge gives a constraint equation which must be satisfied by the independent variables. Although omitted in Algorithm 1, the independent sources can be handled in a similar way to the independent edges. The source currents and/or voltages appear in the constraint equations, which are solved to determine the independent variables.

E_b: set of all the independent edges
E_k: set of all the single-dependent edges
E_u: set of all the constraint edges.

III. PRINCIPAL PARTITION OF 2-GRAPHS

The following 2-graphs, G_{ik} and G_{vk}, are formed from G_i and G_v respectively.

$$G_{ik} \equiv G_i \cdot (E_u \oplus E_e \oplus E_k) \times E_k = G_{ip} \cdot (E_u \oplus E_k) \times E_k \tag{1}$$

$$G_{vk} \equiv G_v \times (E_u \oplus E_j \oplus E_k) \cdot E_k = G_{vp} \times (E_u \oplus E_k) \cdot E_k \tag{2}$$

The principal partition of G_{ik} and G_{vk} results in a partition of E_k into three sets, E_1, E_2 and E_0: E_1 and E_2 are the minimum sets which give

$$\delta_b \equiv -\min_{E_s \subseteq E_k} \{r(G_{vk} \cdot E_s) - r(G_{ik} \times E_s)\} \tag{3}$$

$$\delta_u \equiv -\min_{E_s \subseteq E_k} \{r(G_{ik} \cdot E_s) - r(G_{vk} \times E_s)\} \tag{4}$$

respectively. δ_b and δ_u are called deficiencies.

$E_0 \equiv E_k - E_1 - E_2$.

We can define a pair of trees, T_{ik} and T_{vk} as follows.

T_{ik}, T_{vk}: T_{ik} is a tree of G_{ik} and T_{vk} is a tree of G_{vk} such that T_{ik} and T_{vk} have as many edges as possible in common. (T_{ik} and T_{vk} are called maximally-common trees.)

$\bar{T}_{ik}[\bar{T}_{vk}]$:cotree of $T_{ik}[T_{vk}]$ in $G_{ik}[G_{vk}]$.

Then E_1 and E_2 can be characterized as follows.

Proposition 1. E_1 is the minimum edge set which satisfies (i) $(T_{ik} \cap \bar{T}_{vk}) \subseteq E_1$, (ii) $T_{ik} \cap E_1$ is a forest of $G_{ik} \times E_1$, (iii) $T_{vk} \cap E_1$ is a forest of $G_{vk} \cdot E_1$.

Proposition 2. E_2 is the minimum edge set which satisfies (i) $(T_{vk} \cap \bar{T}_{ik}) \subseteq E_2$, (ii) $T_{vk} \cap E_2$ is a forest of $G_{vk} \times E_2$, (iii) $T_{ik} \cap E_2$ is a forest of $G_{ik} \cdot E_2$.

Actually a pair of trees T_{ik} and T_{vk} and the partition of E_k into E_1, E_2 and E_0 can be obtained at the same time by an algorithm.[8][9] E_0 can be further partitioned into subsets $E_{01}, E_{02}, \ldots, E_{0n}$, and a partial ordering can be given to them. If the second subscripts of the subsets are given in accordance with the partial ordering, then $E_1 \oplus E_{01} \oplus E_{02} \oplus .. \oplus E_m \equiv E_1^*$ for proper m gives δ_b, and $E_2 \oplus E_n \oplus E_{n-1} \oplus .. \oplus E_{m+1} \equiv E_2^*$ gives δ_u; that is,

$$r(G_{vk} \cdot E_1^*) - r(G_{ik} \times E_1^*) = \delta_b \tag{5}$$

$$r(G_{ik} \cdot E_2^*) - r(G_{vk} \times E_2^*) = \delta_u. \tag{6}$$

Now

$$\kappa_b \equiv \min_{E_b \subseteq E_t \subseteq E_p} \{r(G_{vp} \cdot E_t) - r(G_{ip} \times E_t)\} \tag{7}$$

$$\kappa_u \equiv \min_{E_u \subseteq E_t \subseteq E_p} \{r(G_{ip} \cdot E_t) - r(G_{vp} \times E_t)\} \tag{8}$$

are called the electrical connectivities or the nonseparabilities. The relation between κ_b and δ_b can be obtained by noting that $E_t = E_s \oplus E_b$ and that

$$\begin{aligned} \kappa_b &= \min_{E_s \subseteq E_k} \{r(G_{vp} \cdot E_b) + r(G_{vp} \cdot E_t \times E_s) - r(G_{ip} \times E_b) - r(G_{ip} \times E_t \cdot E_s)\} \\ &= r(G_{vp} \cdot E_b) - r(G_{ip} \times E_b) + \min_{E_s \subseteq E_k} \{r(G_{vk} \cdot E_s) - r(G_{ik} \times E_s)\}. \end{aligned} \tag{9}$$

Then we have

$$\kappa_b = \rho_b - \delta_b \tag{10}$$

where

$$\rho_b \equiv r(G_{vp} \cdot E_b) - r(G_{ip} \times E_b). \tag{11}$$

Dually we have

$$\kappa_u = \rho_u - \delta_u \tag{12}$$

where

$$\rho_u \equiv r(G_{ip} \cdot E_u) - r(G_{vp} \times E_b). \tag{13}$$

If we denote the minimum edge sets giving κ_b and κ_u by E_c and

E_w respectively, then we have

$$E_c = E_b \oplus E_1 \,, \tag{14}$$

$$E_w = E_u \oplus E_2 . \tag{15}$$

$E_c^* \equiv E_b \oplus E_1^*$ and $E_w^* \equiv E_u \oplus E_2^*$ are edge sets giving κ_b and κ_u respectively.

Let

$$G_{i0} \equiv G_{ik} \times (E_0 \oplus E_1) \cdot E_0$$

$$G_{v0} \equiv G_{vk} \cdot (E_0 \oplus E_1) \times E_0 .$$

Then G_{i0} and G_{v0} have a common tree, which is denoted by T_0.

$$T_0 = \bigoplus_{m=1}^{n} T_{0m} \tag{16}$$

where T_{0m} is a common tree of G_{i0m} and G_{v0m}:

$$G_{i0m} \equiv G_{i0} \times (E_{01} \oplus E_{02} \oplus \dots \oplus E_{0m}) \cdot E_{0m}$$

$$G_{v0m} \equiv G_{v0} \cdot (E_{01} \oplus E_{02} \oplus \dots \oplus E_{0m}) \times E_{0m} .$$

The decomposition of G_{i0} and G_{v0} into G_{i0m} and G_{v0m} $(m=1,2,..n)$ is called the fine decomposition. An elementary common-tree transformation is possible within G_{i0m} and G_{v0m} only, that is, no common tree can be obtained from T_0 by exchanging an edge in E_{0m} with that in E_{0l} $(l \neq m)$.

IV. SOLUTION TO THE DIAGNOSIS PROBLEM

From the definition of E_1 and E_2,

$$\delta_b = -r(G_{vk} \cdot E_1) + r(G_{ik} \times E_1) = r(G_{ik} \times E_1) + n(G_{vk} \cdot E_1) - |E_1| \tag{17}$$

$$\delta_u = -r(G_{ik} \cdot E_2) + r(G_{vk} \times E_2) = |E_2| - r(G_{ik} \cdot E_2) - n(G_{vk} \times E_2) . \tag{18}$$

Now $r(G_{ik} \times E_1) + n(G_{vk} \cdot E_1)$ is the number of equations obtained by use of KCL and KVL for the unkown variables associated with E_1. $|E_1|$ is the number of unknown variables. Thus δ_b is the number of excess equations to determine the unknown variables associated with the edges of E_1. Dually δ_u is the number of equations which are short of to determine the unknown variables associated with the edges of E_2. Even if $\delta_b>0$, the equations for E_1 must be consistent, since we have assumed that the original network has a unique solution. Only a part of the equations for E_1 need be solved. On the other hand, if $\delta_u>0$, then the unknown variables associated with the edges of E_2 can not be determined.

The current[voltage] of an edge in E_u or E_e[E_u or E_j] must be determined, if possible, by use of KCL[KVL] only, since Ohm's law cannot be used for it. It must be a bridge[self-loop] in $G_i\cdot(E_u\oplus E_e\oplus E_2)$ [$G_v x(E_u\oplus E_j\oplus E_2)$].

E_{id} : set of bridges in $G_i\cdot(E_u\oplus E_e\oplus E_2)$

E_{vd} : set of self-loops in $G_v x(E_u\oplus E_j\oplus E_2)$

For an edge in E_{id}[E_{vd}] there is a cutset in G_i[tieset in G_v] which consists of the edge and those in $E_b\oplus E_j\oplus E_0\oplus E_1$ [$E_b\oplus E_e\oplus E_0\oplus E_1$], and the current[voltage] of the edge can be determined by KCL[KVL] applied to the cutset[tieset]. Thus we have:

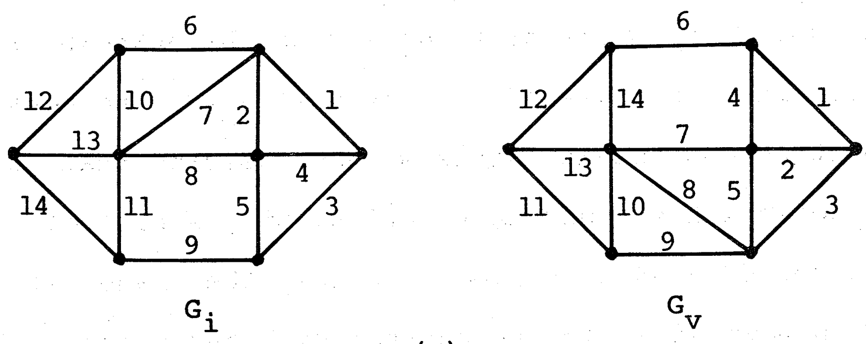
Theorem 1. The network is diagnosable if and only if

$$F_i \subseteq E_0\oplus E_1\oplus E_{id} \tag{19}$$

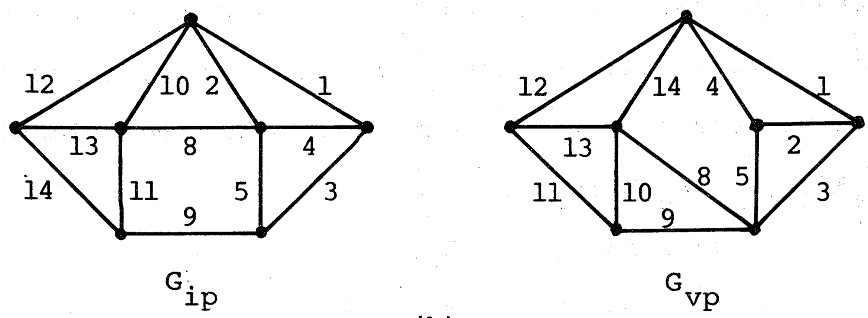
and

$$F_v \subseteq E_0\oplus E_1\oplus E_{vd} \ . \tag{20}$$

Example 1. An Example is given in Fig.1. From G_i and G_2 shown in Fig.1(a), G_{ip} and G_{vp} in Fig.1(b) are derived. G_{ik} and G_{vk} are shown in Fig.1(c). From the principal partition of G_{ik} and G_{vk}, we get E_1={4}, E_2={11} and E_0={5,8,9,10}. E_0 is further



(a)



(b)

Fig.1. Example 1. (a) G_i and G_v. $E_b=\{1,2,3\}$, $E_j=\{7\}$, $E_e=\{6\}$, $E_b=\{4,5,8,9,10,11\}$ and $E_u=\{12,13,14\}$. (b) G_{ip} and G_{vp}.

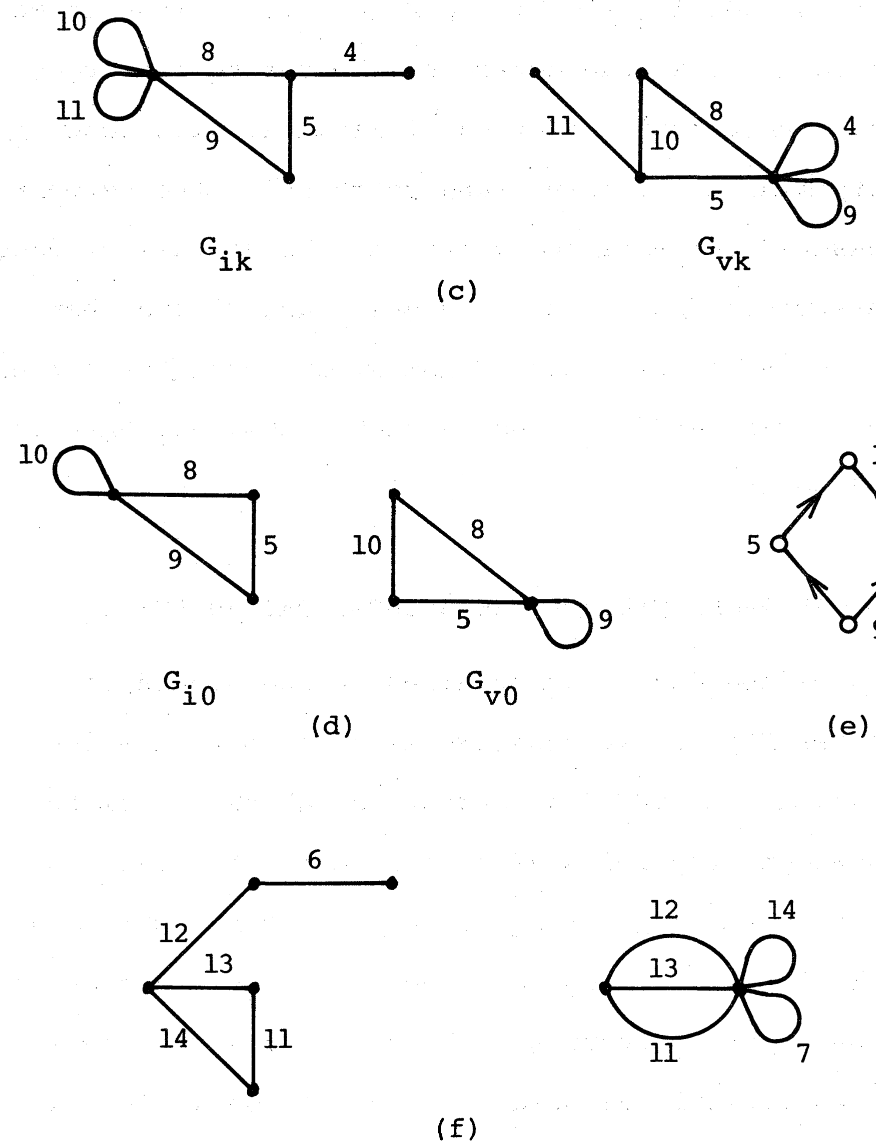


Fig.1. Example 1(continued) (c) G_{ik} and G_{vk}. (d) G_{i0} and G_{v0}. (e) partial ordering of the edges in E_0. (f) $G_i \cdot (E_u \oplus E_e \oplus E_2)$ and $G_v x (E_u \oplus E_j \oplus E_2)$

partitioned into $E_{01}=\{9\}$, $E_{02}=\{5\}$, $E_{03}=\{8\}$ and $E_{04}=\{10\}$, and a partial ordering can be given to them as shown in Fig.1(e). From Fig.1(f) we get $E_{id}=\{6,12\}$ and $E_{vd}=\{7,14\}$. Thus if we are given F_i and F_v, we can determine the diagnosability of the network. The partial ordering of Fig.1(e) also shows the order to determine the currents and voltages in the network from the measured currents and voltages: The voltage of edge 9 is first determined from those of edges 2 and 3, and then its current is obtained by use of Ohm's law. Next the current of edge 5 or 8 can be determined by use of KCL, and then the voltage, and so on.

V. TOPOLOGICAL PROPERTIES OF THE SEQUENTIOAL ANALYSIS

For a single-dependent edge obtained in the sequential process, either KCL or KVL equation is used to relate its current or voltage to the independent variables. Thus the number of unknown variables associated with the edges in E_k is equal to the number of equations for them. (Ohm's law is used to relate the current and the voltage of an edge, and thus only one of the current and the voltage is considered to be the unknown variable associated with the edge.) Thus we have the following theorem from eqs.(17) and (18).

Theorem 2. For the principal partition of G_{ik} and G_{vk} derived from the sequential process,

$$\delta_b=0,\ \delta_u=0,\ E_1=\emptyset,\ E_2=\emptyset \text{ and } E_0=E_k. \tag{21}$$

Now let

$e_{s1}, e_{s2}, .., e_{sn}$ $(n=|E_k|)$: single-dependent edges covered in this order in the sequential process,

$E_{s1} \equiv \{e_{s1}\}$, $E_{sm} \equiv E_{sm-1} \oplus \{e_{sm}\}$ $(m=2,3,..,n)$ $(E_{sn}=E_k)$.

Then each of $E_{sm}(m=1,2,..,n)$ gives $\delta_b=0$, and thus $\{e_{sm}\}$ $(m=1,2,..,n)$ must be the subsets of E_0 defined with respect to the principal partition of G_{ik} and G_{vk}(which are now equal to G_{i0} and G_{v0} respectively). The order of $e_{sm}(m=1,2,..,n)$ is in accordance with the partial ordering of the subsets of E_0 (The order of $e_{sm}(m=1,2,..,n)$ may not be unique, since more than one edge may become current or voltage dependent to E_β, and then an edge can be arbitrarily chosen next to cover.).

<u>Theorem 2(continued)</u> Each of the subsets $E_{01}, E_{02},.., E_{0n}$ of E_0 consists of exactly one edge, and $n=|E_k|$. They can be set as $E_{0m}=\{e_{sm}\}(m=1,2,..,n)$. Each of $E_{sm}(m=1,2,..,n)$ gives $\delta_b=0$. A necessary and sufficient condition for this to hold is that there exists exactly one common tree of G_{ik} and G_{vk}.

Next $E_b[E_u]$ contains neither cutset in $G_{ip}[G_{vp}]$ nor tieset in $G_{vp}[G_{ip}]$. Thus we have the following theorem.

<u>Theorem 3.</u>

$$\rho_b=|E_b|, \quad \rho_u=|E_u|, \quad \rho_b=\rho_u. \tag{22}$$

$$\kappa_b=\kappa_u=|E_b|, \tag{23}$$

and each of $E_{tm} \equiv E_{sm} \oplus E_b(m=1,2,..,n)$ gives κ_b.

<u>Example 2.</u> An example is given in Fig.2. The edges of G_i and G_v in Fig.2(a) are numbered according to the sequential process. $E_j=\{j\}$ and $E_e=\{e\}$. We get G_{ip} and G_{vp} as shown in Fig.2 (b), and then G_{ik} and G_{vk} , in Fig.2(c). The partial ordering of edges is given in Fig.2(d). It can be easily seen that

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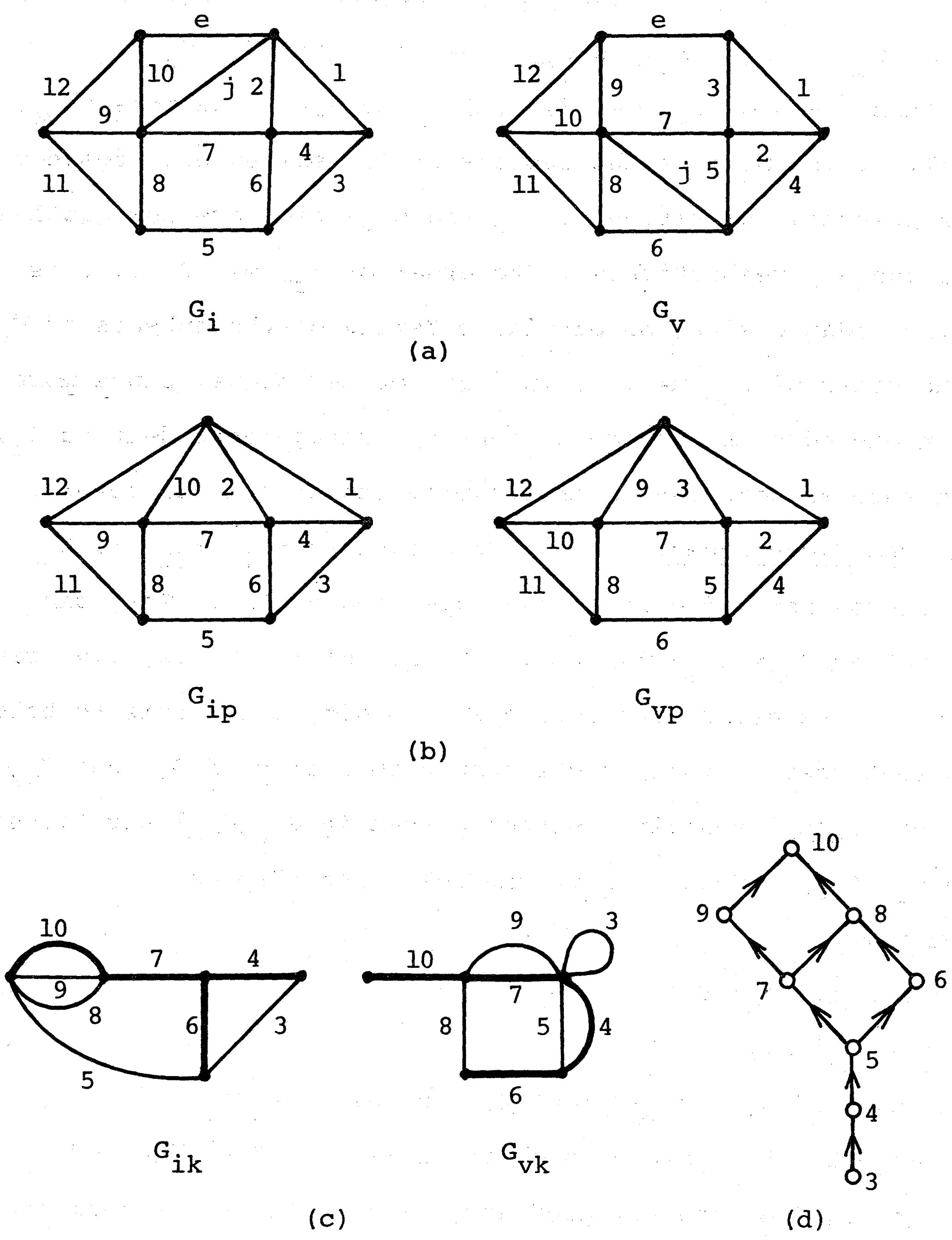

Fig.2. Example 2. (a) G_i and G_v. $E_j=\{j\}$, $E_e=\{e\}$.
(b) G_{ip} and G_{vp}. (c) G_{ik} and G_{vk}. (d) partial ordering of the edges in $E_0=E_k$.

there exists exactly one common tree of G_{ik} and G_{vk}. It is indicated by the thick lines in Fig.2(c). It consists of the current dependent edges, and its cotree, of voltage dependent edges. The edges of the common tree and those of $E_u \oplus E_e [E_b \oplus E_e]$ form a tree of $G_i [G_v]$. These trees of G_i and G_v are called a linkage pair of trees.

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